16

Principles of Electromechanical Systems

In this chapter, we lead you through a study of the mathematics and physics of electrical machines. After completing the chapter, you should be able to

- Review the basic principles of electricity and magnetism.
- Understand the concepts of reluctance and magnetic circuits.
- Understand the properties of magnetic materials.
- Understand the principles of transformers.
- Calculate values of voltage, current, and turns for transformers using simple formulas
- Draw the equivalent circuit for a real transformer and find its parameters.
- Define reactive power and apparent power.
- Understand the basic concepts of electromagnetomechanical systems.
- Identify electrical devices in an everyday setting and be able to describe their basic operating characteristics.

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16.1 ELECTRICITY AND MAGNETISM

A connection between electricity and magnetism was discovered accidentally by H. C. Orsted, a Danish physicist, over 100 years ago, who noticed that a compass needle is deflected when brought into the vicinity of a current carrying wire. Thus, currents induce in their vicinity magnetic fields.

As explained in Chapter 8, Faraday, who found that changing magnetic fields though loops of wire could cause currents to be induced, discovered a further connection between electricity and magnetism.

Magnetism is one of the most phenomena in the electrical field. It is the force used to produce most of the electrical power in the world. The term magnetism is derived from Magnesia, the name of a region in Asia where lodestone, a naturally magnetic iron ore, was found in ancient times. The ancient Greeks and also the early Chinese knew about strange and rare stones with the power to attract iron. A steel needle stroked with such a "lodestone" became "magnetic" as well and later the Chinese found that such a needle, when freely suspended, pointed north south. The magnetic compass soon spread to Europe. Columbus used it when he crossed the Atlantic Ocean, noting not only that the needle deviated slightly from exact north (as indicated by the stars) but also that the deviation changed during the voyage. Around 1600 William Gilbert, physician to Queen Elizabeth I of England, proposed an explanation: the Earth itself was a giant magnet, with its magnetic poles some distance away from its geographic ones.

It is evident from Figure 16-1 that in the upper (northern) half of the earth, the magnetic field is directed toward the earth; in the lower (southern) half, the field is directed away from the earth.

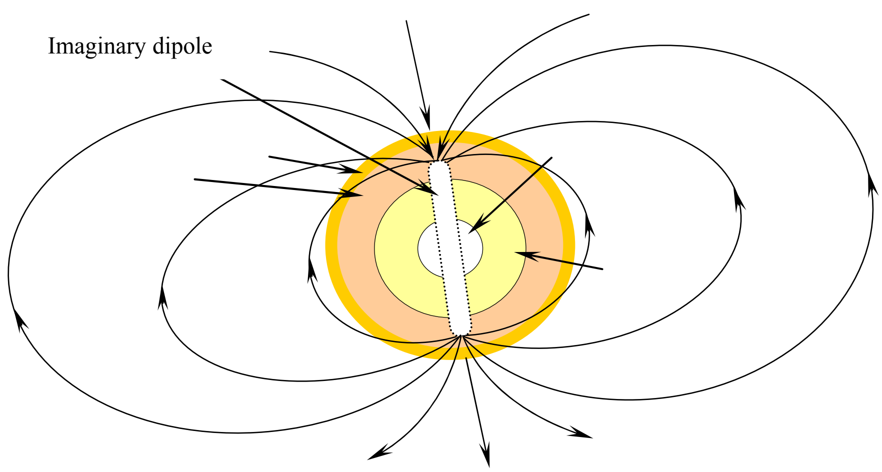


Figure 16-1 Earth may be thought of as a dipole magnet.

Magnetic fields are the fundamental mechanisms by which energy is converted from one form to another such as in transformers, motors, and generators. The following principles describe how magnetic fields are used in the above devices:

1. A current-carrying conductor produces a magnetic field in the area around it.
2. A time-changing magnetic field induces a voltage in a coil of wire if it passes through the coil. (This is the function of a transformer.)
3. A current-carrying wire in the presence of a magnetic field has a force induced on it. (This is the function of a motor-Chapter 17.)
4. A moving wire in the presence of a magnetic field has a voltage induced in it. (This is the function of a generator-Chapter 17.)

16.2 MECHANICAL WORK AND POWER

16.2.1 Work

Work (W) is the energy used or gained. For linear motion, work is the application of a force (F) through a distance (r) assuming that the force is collinear with the direction of motion

$$W = \int F\, dr \tag{16.1}$$

For the special case of a constant force applied collinear with the direction of motion, Equation (16.1) becomes

$$W = F \times r \tag{16.2}$$

The units of work are joules in SI. For rotational motion, work is the application of a torque through the angular position (θ) (angle at which object is oriented)

$$W = \int T\, d\theta \tag{16.3}$$

For the special case of a constant torque, Equation (16.3) becomes

$$W = T\, \theta \tag{16.4}$$

16.2.2 Power

Power (P) is the rate of doing work. It may be defined as

$$P = \frac{dW}{dt} \tag{16.5}$$

For the special case of constant work, power is

$$P = \frac{W}{t} \tag{16.6}$$

Power is measured in joules per second (watts). For the special case of a constant force, which is collinear with the direction of motion, power is

$$P = \frac{dW}{dt} = \frac{d}{dt}(Fr) = F\left(\frac{dr}{dt}\right) = Fv \tag{16.7}$$

where v is one-dimensional linear velocity defined as

$$v = \frac{dr}{dt} \tag{16.8}$$

For rotational motion, assuming constant torque, we write

$$P = \frac{dW}{dt} = \frac{d}{dt}(T\theta) = T\left(\frac{d\theta}{dT}\right) = T\omega \tag{16.9}$$

where ω is the angular velocity

$$\omega = \frac{d\theta}{dt} \tag{16.10}$$

Example 16-1

If an 80-kg person climb to the tope of a building (10 m) during a time of 30 seconds. Find the work and power.

Solution: In the MKS system of measure, each kg of mass exerts 9.8 newtons of force at the earth surface. Using Equation (16.2), we get

$$W = Fr$$

$$= 80\text{ kg} \times \frac{9.8\text{ N}}{\text{kg}} \times 10\text{ m} = 7.84\text{ kNm}$$

A newton-meter is a joule

$$W = 7.84\text{ kJ}$$

To calculate the power, use Equation (16.6)

$$P = \frac{7.84\text{ kJ}}{30\text{s}} = 261.33\,\frac{\text{J}}{\text{s}}$$

16.3 ELECTRICAL POWER

16.3.1 Sinusoidal Power Equation

We knew from the previous section that

	$P = \frac{W}{t}$	

Since

	$i = \frac{Q}{t}$	

then,

	$P = \frac{W}{t} = \frac{W}{Q} \times \frac{Q}{t}$ $= v \times i$	(16.11)

The voltage is used as the reference, assigning it a phase angle of 0^o.

	$v = V_P \sin(wt)$	(16.12)

The current may be leading or lagging this voltage

	$i = I_P \sin(wt + \theta)$	(16.13)

Instantaneous power is the product of v and I in Equation (16.6) and Equation (16.13)

$$P = V_P I_P \sin(wt)\sin(wt+\theta) \tag{16.14}$$

We may simplify the product of two different sine functions

$$\sin(X)\sin(Y)$$

Applying a trig identity, with little algebra simplification

$$P = \frac{V_P I_P}{2}\cos\theta\left[1-\cos(2wt)\right] + \frac{V_P I_P}{2}\sin\theta\sin(2wt) \tag{16.15}$$

RMS values are generally used when working with power

$$P = V_{rms} I_{rms}\cos\theta\left[1-\cos(2wt)\right] + V_{rms} I_{rms}\sin\theta\sin(2wt) \tag{16.16}$$

It is evident from Equation (16.16) that the power has both DC and AC since $\sin\theta$ and $\cos\,\theta$ terms are fixed quantities. The sinusoidal part is at $2\omega t$, twice the frequency of the applied voltage.

$$P = \underbrace{V_{rms} I_{rms}\cos\theta}_{\text{DC}}\,\underbrace{\left[1-\cos(2\omega t)\right] + V_{rms} I_{rms}\sin\theta\sin(2\omega t)}_{\text{AC}} \tag{16.17}$$

16.4 MAGNETIC CIRCUITS

The concept of magnetic circuit is useful in estimating the flux produced by coils wound in ferromagnetic material. A magnetic circuit may be compared with an electric current in which EMF, or voltage, produces a current flow. The behavior of magnetic circuit is governed by equations analogous to those of a DC electric circuit. Calculations of excitation are usually based on Ampere's law, given as

$$\oint \bar{H}.\bar{dl} = \text{current enclosed (ampere - turns enclosed)} \tag{16.18}$$

which is valid for steady currents and is approximately valid when the time variation of current is so slow that electromagnetic radiation is negligible. The magnetomotive force (MMF), or the magnetic potential difference that tends to produce a magnetic field, has the unit of ampere-turns. It will produce a magnetic flux as shown in Figure 16-2. The MMF can be compared with EMF, the flux (ϕ) can be compared to current (I), and the reluctance ($\Re$) of magnetic field is the counterpart of electrical resistance (R)

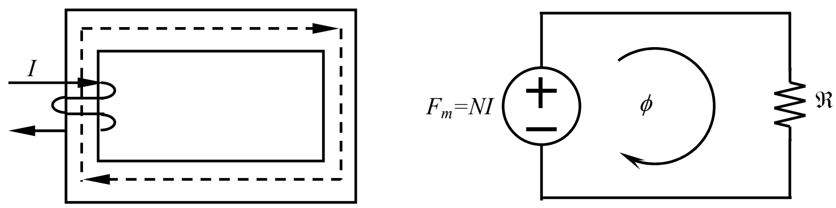


Figure 16-2 Analogy between magnetic and electric circuits.

$$\phi = \frac{F}{\Re} \tag{16.19}$$

Where, ϕ = magnetic flux of circuit, measured in weber (Wb).
F = magnetomotive force of circuit, measured in ampere-turn.
$\Re$ = reluctance of a circuit, measured in ampere-turn per weber (Wb).

The reluctance is represented by the following formula

$$\Re = \frac{l}{\mu A} \tag{16.20}$$

where l is the mean path length of the core measured in meters, A is the cross-sectional area of the core, and μ is the magnetic permeability of material. Reluctances in a magnetic circuit obey the same rules as resistances in an electric circuit. For example, the reluctance of a number of reluctances in series is given by

$$\Re_{eq} = \Re_1 + \Re_2 + \Re_3 + .. + \Re_n \tag{16.21}$$

and for reluctances in parallel

$$\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \frac{1}{\Re_3} + .. + \frac{1}{\Re_n} \tag{16.22}$$

If there is an air gap in the flux path in a core, the effective cross-sectional area of the air gap will be larger than the cross-sectional area of the iron core on either side. The extra effective area is caused by the "fringing effect" of the magnetic field at the air gap as shown in Figure 16-3. Fringing occurs because the reluctances of different paths available near the air gap are quite comparable to each other, and the flux lines spread out. The effect of fringing increases with the length of the air gap. The effect of fringing is taken into consideration in magnetic circuit calculations by recognizing that the effective area of the air gap is greater than the actual area. However, the effect may be ignored when the air gap is small.

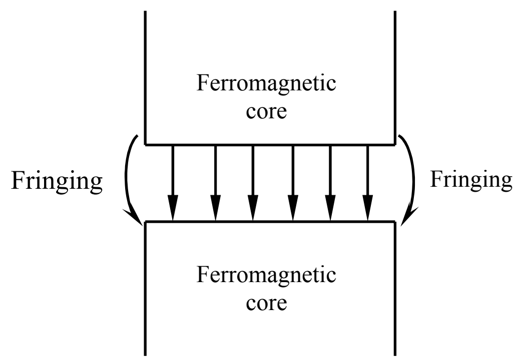


Figure 16-3 An air gap in the flux path in a core.

Table 16-1 provides an analogy between electric circuits and magnetic circuits.

Table 16-1 Analogy between Electric and Magnetic Circuits

Electrical Quantity	**Magnetic Quantity**
Electric field intensity E, V/m	Magnetic field intensity H, A-turns/m
Voltage v, V	Magnetomotive force F_m, A-turn
Current i, I	Magnetic flux ϕ, Wb
Current density J, A/m^2	Magnetic flux density B, Wb/m2
Resistance R, Ω	Reluctance $\Re$, A-turn/Wb
Conductivity σ, 1/Ω-m	Permeability μ, Wb/A-m

Example 16-2

A coil has MMF of 500 A-turn and a reluctance of 2×10^6 A-turn/Wb. Find the total flux.

Solution: Use Equation (16.19)

$$\phi = \frac{F_m}{\Re} = \frac{500 \text{ A - turn}}{2 \times 10^6 \text{ A - turn/Wb}} = 2.5 \times 10^{-5} \text{ Wb}$$

Example 16-3

Consider the magnetic circuit in Figure 16-4. Steady currents I_1 and I_2 flow in two coils of N_1 and N_2 turns, respectively on the outside legs of the ferromagnetic material. The core has a cross-sectional area A and a permeability μ. Determine the magnetic flux in the center leg.

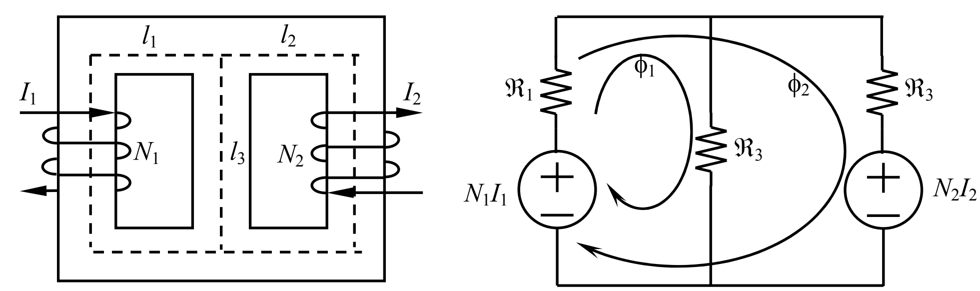


Figure 16-4 (a) Magnetic circuit. (b) Analogous circuit.

Solution: The equivalent magnetic circuit for loop analysis is shown in 16-4 (b). We observe two sources of MMF, N_1I_1, and N_2I_2 and three reluctances $\Re_1$, $\Re_2$, and $\Re_3$. Since we are going to find the magnetic flux in the center leg AB, it is required to consider a two-loop network with one loop flux (ϕ_1) flows through the center leg. The reluctances are computed on the basis of average path lengths. This can be stated as

$$\Re_1 = \frac{l_1}{\mu A}$$

$$\Re_2 = \frac{l_2}{\mu A}$$

$$\Re_3 = \frac{l_3}{\mu A}$$

Now, we write the two loop equations

$$N_1 I_1 = \phi_1 (\Re_1 + \Re_2) + \Re_1 \phi_2$$
$$N_1 I_1 - N_2 I_2 = \phi_2 (\Re_1 + \Re_2) + \Re_1 \phi_1$$

Solving these two simultaneous equations, we obtain

$$\phi_1 = \frac{N_1 I_1 \Re_2 + N_2 I_2 \Re_1}{\Re_1 \Re_2 + \Re_1 \Re_2 + \Re_1 \Re_2}$$

16.5 MAGNETIC MATERIALS

All materials show some magnetic effects. With the exception of ferromagnetic materials these effects are weak. Depending on their magnetic behavior, materials may be classified according to some of their basic magnetic properties, particularly whether or not they are magnetic and how they behave in the vicinity of an external magnetic field.

16.5.1 Non-magnetic Materials

Most materials we encounter have no obvious magnetic properties - they are said to be non-magnetic. In these materials, the magnetic fields of the individual atoms are randomly aligned and thus tend to cancel out, as shown in Figure 16-5. The relative permeability of such materials is relatively constant (around 1, more or less) and independent of the applied field. Examples of nonmagnetic materials are copper, brass, silver, water, air, aluminum, and biological tissues.

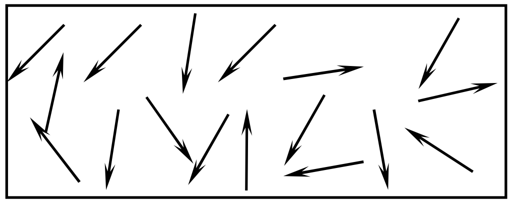


Figure 16-5 Non-magnetic material.

16.5.2 Permanent Magnetic Materials

Permanent magnets are magnets that do not require any power or force to maintain their field. The basic purpose of any magnet is to store energy or to convert energy from one form to another. In a permanent magnet, however, the magnetic fields of the individual atom are aligned in one preferred direction, giving rise to a net magnetic field, as shown in Figure 16-6.

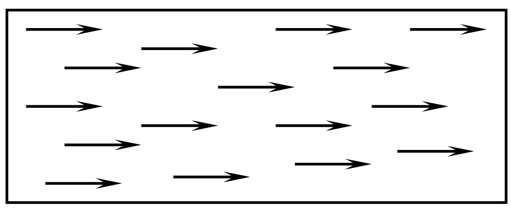
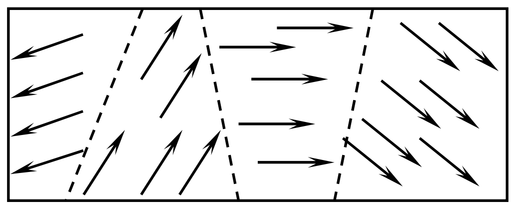


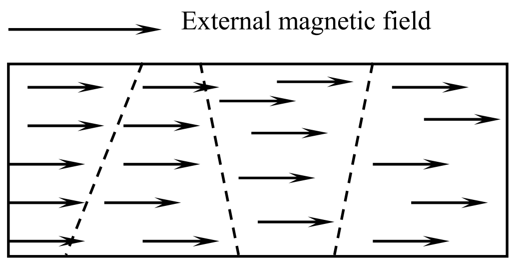
Figure 16-6 Permanent magnet.

16.5.3 Ferromagnetic Materials

In this material, there are domains in which the magnetic fields of the individual atoms align, but the orientation of the magnetic fields of the domains is random, giving rise to no net magnetic field. This is illustrated in Figure 16-7 (a). A significant property of ferromagnetic materials is that when an external magnetic field is applied to them, the magnetic fields of the individual domains tend to line up in the direction of this external field. This is because of the nature of magnetic forces, which causes the external magnetic field to be enhanced. This is illustrated in Figure 16-7 (b). The relative permeability of ferromagnetic materials varies over a wide range for various applied fields. Examples of these materials are iron, nickel, cobalt, and mumetal.

The advantage of using a ferromagnetic material for cores in electric machines and transformers is that of getting more flux for a given MMF with iron than with air. Another area where ferromagnetic materials are employed is in magnetic recording devices, such as for cassette tapes, floppy discs for computers, and the magnetic stripe on the back of credit cards. These devices essentially take information in the form of electrical signals and permanently encode it into a magnetic material.


(a)



(b)

Figure 16-7 Ferromagnetic material.

Figure 16-4

Determine the relative permeability of the typical ferromagnetic material when B = 0.5 Tesla and H = 50 A-turns/m.

Solution: We know from Chapter 14 that the permeability of a material is given by

$$\mu = \frac{B}{H} = \frac{0.5}{50} = 0.01\,\text{H/m}$$

and the relative permeability is given by

$$\mu_r = \frac{\mu}{\mu_o} = \frac{0.01}{4\pi \times 10^{-7}} = 7957.74$$

16.6 FARADAY'S LAW

In 1831, Michael Faraday in London found that a magnetic field could produce current in a closed circuit when the magnetic flux linking the circuit is changing. This phenomenon is known as *electromagnetic induction*. Faraday concluded from his experiment that the induced current was proportional, not to the magnetic flux itself, but to its rate of change.
Consider the closed wire loop shown in Figure 16-8. A magnetic field with magnetic flux density **B** is normal to the plane of the loop. If the direction of **B** is upward and decreasing in value, a current *I* will be generated in the upward direction. If **B** is directed upward but its value is increasing in magnitude, the direction of the current will be opposite.

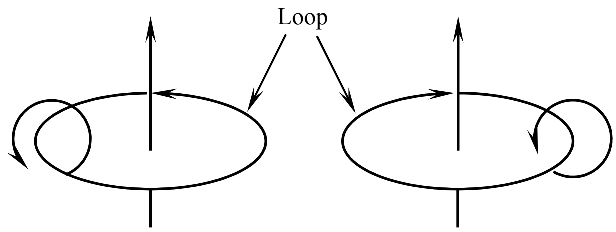


Figure 16-8 Induced currents due to magnetic flux density **B**.

When **B** is decreasing, the current induced in the loop is in such a direction as to produce a field, which tends to increase **B** [Figure 16-8 (a)]. However, when **B** is increasing, the current induced in the loop is in such a direction as to produce a field opposing **B** [Figure 16-8 (b)]. Therefore, the induced current in the loop is always in such a direction as to produce flux opposing the change in **B**. This phenomenon is called *Lenz's law*. As the magnetic field changes, it produces an **E** field. Integrating **E** field around a loop yields an *electromotive force*, or *e*, measured in volts as follows

$$e = \int \mathbf{E} \cdot d\mathbf{l} \qquad (16.23)$$

e appears between the two terminals, if the loop is open circuit. This is the basic for the operation of an electric generator.

A quantitative relation between the EM force induced in a closed loop and the magnetic field producing e can be developed. This is represented by

$$e = -\frac{d\phi}{dt} \tag{16.24}$$

where $\phi = \iint \mathbf{B} \cdot d\mathrm{s}$ is the total flux in webers (Wb). If a coil has N turns and if the same flux passes through all of them, then we write Equation (16.24) as

$$e = -N\frac{d\varphi}{dt} \tag{16.25}$$

Equation (16.25) may be written as

$$e = -\frac{d}{dt}\iint \mathbf{B} \cdot ds \tag{16.26}$$

where ds is surface element measured in square meter (m^2) and t is time measured in seconds (s).

Although Joseph Henry in Albany, New York also discovered the result shown in Equation (16.25), the credit is still attributed to Faraday. Both Faraday and Henry discovered the above finding independently at about the same time, however, it is known as *Faraday's law* of induction. Faraday's law is well known through its importance in motors, generators, transformers, induction heaters, and other similar devices. Also, Faraday's law provides the foundation for the electromagnetic theory.

The total time derivative in Equation (16.26) operates on **B**, as well as the differential surface area ds. Therefore, e can be generated under three conditions: a time-varying magnetic field linking a stationary loop; a moving loop with a time-varying area; and a moving loop in a time-varying magnetic field.

Example 16-5

Consider a coil of wire wrapped around an iron core. If the flux in the core is given by the equation

$$\phi = 0.5\sin\omega t \text{ Wb}$$

and if there are 200 turns on the core, what voltage is produced at the terminals of the coil?

Solution: Apply Equation (16.25)

$$e = -N\frac{d\phi}{dt} = -100\frac{d}{dt}\left(0.5\sin\omega t\right)$$
$$= -50\cos\omega t$$

16.7 TRANSFORMERS

A transformer is a static device in which two or more stationary electric circuits are magnetically coupled. It changes AC electric power at one voltage or current level to AC electric power at another voltage or current level through the action of a magnetic field. The principle of transformer is based on Faraday's law: given two magnetically coupled coils, a changing current in one coil will induce an electromotive force in the other one. Such induced force is called *transformer voltage* and the arrangement of coils for such purpose is called a *transformer*.

Transformers are used to raise or lower voltage in AC transmission and distribution power systems; to isolate one electric circuit from another; to provide reduced voltage in power supply circuits that provide DC to most of the consumer appliances.

The principle of transformer action is applicable in many ways to motors and generators. For example, an induction motor is called a rotating transformer since their operation is almost identical. We will study the details of motors and generators in Chapter 17.

16.7.2 Operating Principles

Conventional transformers have two windings coupled magnetically through a common ferromagnetic core. However, *autotransformers* have only one winding, and still others-*multiwinding transformers*- have more than two windings. These windings are usually separated from each other. They have a common magnetic flux present within the core.

Core

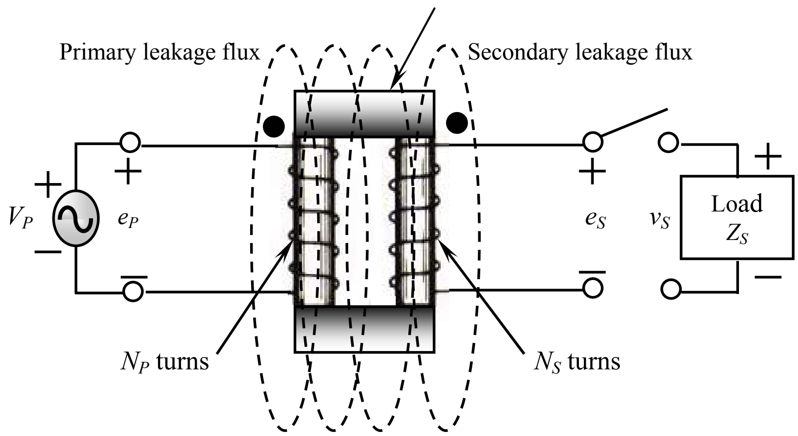


Figure 16-9 A simple transformer.

One of the coils in the conventional transformer is called *primary winding*, which is connected to the input AC power supply. A second coil is called *secondary winding*, which supplies electric power to the driven load. In Figure 16-9, one winding of the transformer has been connected to an AC supply, and the other winding has been connected to a load. As current increases from zero to its peak positive point, a magnetic field expands outward around the coil. When the current decreases from its peak point toward zero, the magnetic field collapses. When the current increases toward its negative peak, the magnetic field again expands, but with an opposite polarity of the previous one. The field once again collapses when the current decreases from its negative peak toward zero. This regular expanding and collapsing of the magnetic field cuts the windings of the primary and induces a voltage into it. The induced voltage opposes the applied voltage and limits the current flow of the primary. When a coil induces a voltage into itself, it is known as *self- induction*.

Another term that we should be familiars with that plays an important role in understanding transformers is called *mutual inductance*. Mutual inductance is when you take two coils, apply current to one coil only, do not matter which one as long as it is AC current, and then place close together both coils (as long as they do not electrically touch each other) and mutual inductance will take place. This is where the expanding and collapsing flux magnetic fields of the first coil with the current will cut across the winding of the second coil without the current, and voltage will be induced in the second coil.

In order to restrict most of the flux to a defined path linking the windings, the core is usually made of ferromagnetic material. In order to reduce the losses caused by eddy currents in the core, the core is usually comprised of a stack of thin laminations.

16.7.3 Ideal Transformer

If a time-varying voltage source is connected to the primary winding, then by virtue of Faraday's law, a corresponding time-varying flux $d\phi/dt$ is established and a counter (induced) voltage e_P is developed in the primary coil L_P

$$v_P = e_P = N_P \frac{d\phi}{dt} \qquad (16.27)$$

Where N_P is the primary winding turns. Remember here that the resistance drop in the winding is neglected, so the counter voltage e_P equals the applied voltage at the primary winding v_P.

As seen from Figure 16-8, the flux links also coil L_S and a counter voltage e_S is induced across the output coil

$$v_S = e_S = N_S \frac{d\phi}{dt} \qquad (16.28)$$

Where N_S is the secondary winding turns. A basic law concerning transformers is that all values of a transformer are proportional to its *turns ratio*. Turns ratio is the ratio of the voltages across the primary and secondary windings of an ideal transformer. By combining Equation (16.27) and Equation (16.28), we will get the turns ratio

$$\frac{v_P}{v_S} = \frac{N_P}{N_S} = a \qquad (16.29)$$

Where a is the turns ratio. Let us now consider the case in which the waveforms of the applied voltage and flux are sinusoidal. In this case, the flux is given by

$$\phi = \phi_{\max} \sin \omega t \qquad (16.30)$$

Where $\phi_{\max}$ is the maximum value of the flux, and ω is $2\pi f$. Accordingly, Equation (16.30) is written as

$$e_P = N_P \frac{d\phi}{dt} = \omega N_P \phi_{\max} \cos \omega t \qquad (16.31)$$

The RMS value of the induced voltage is given by

	$E_P = \frac{2\pi}{\sqrt{2}} f N_P \varphi_{\max} = 4.44 f N_P \varphi_{\max}$	(16.32)

Equation (16.32) can be written as

	$\phi_{\max} = \frac{V_P}{4.44 f N_P}$	(16.33)

Where V_P is the RMS value of the applied voltage with the assumption that $e_P = V_P$. Equation (16.33) shows that the flux is determined by the applied voltage, the frequency of operation, and number of turns in the winding.

If we connect the load to the circuit, current i_S will flow and a corresponding MMF is produced ($F_m = N_S\, i_S$). This MMF would cause the flux in the core to change; however, this is not possible, since a change in ϕ would cause a corresponding change in the voltage induced across the input coil. But this voltage is determined (fixed) by the source v_I (and is therefore $d\phi / dt$), so that the input coil is forced to generate a counter MMF to oppose the MMF of the output coil. This is accomplished as the input coil draws a current i_P from the source v_P such that

	$i_P N_P = i_S N_S$ or $\frac{i_S}{i_P} = \frac{N_P}{N_S}$	(16.34)

Multiplication of Equation (16.33) and Equation (16.34) gives an equation that shows the conservation of apparent power in the ideal transformer. Division of Equation (16.33) and Equation (16.34) yields

	$Z_L^P = \frac{V_P}{I_P} = \left(\frac{N_P}{N_S}\right)^2 \frac{V_S}{I_S} = \left(\frac{N_P}{N_S}\right)^2 Z_L$	(16.35)

where Z_L^P is the load impedance referred to the primary side and Z_L is the impedance of the load in the secondary circuit. Figure 16-10 shows the equivalent circuit viewed from the primary winding.

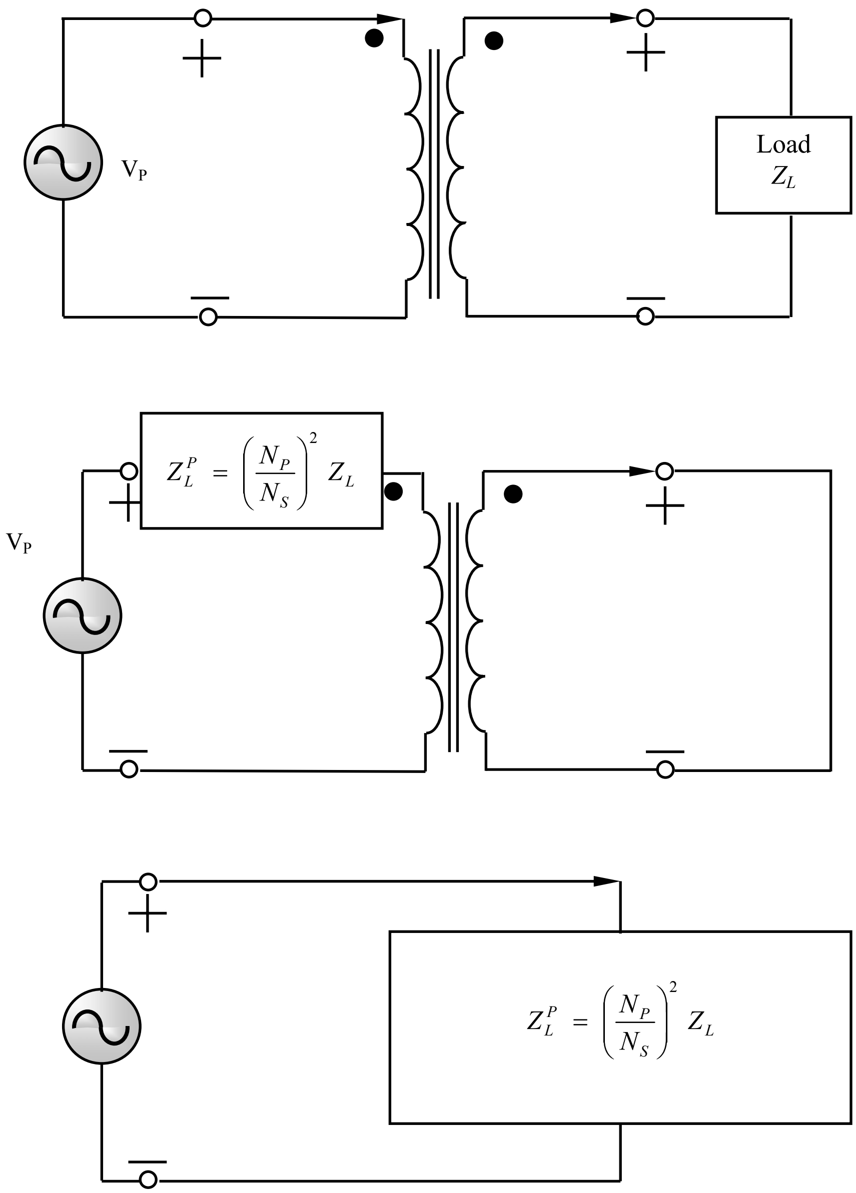


Figure 16-10 Equivalent circuit viewed from the primary winding.

Example 16-6

A step-down transformer has 300 turns of wire on the primary and 60 turns of wire on secondary. This is a ratio of 5:1 (300/60 = 5). Assume that 120 V is connected to the primary winding. What is the voltage of the secondary winding?

Solution: Use Equation (16.29) to find the voltage of the secondary winding

$$\frac{v_P}{v_S} = \frac{N_P}{N_S}$$

$$\frac{120}{v_S} = \frac{300}{60}$$

$$v_S = 24\text{ V}$$

Example 16-7

Assume that a load of 5 Ω is connected to secondary winding of the transformer of Example 16-6. Calculate the current flow in the secondary and primary windings.

Solution: The current flow of the secondary can be computed using Ohm's law since the voltage and impedance

$$I = \frac{\text{V}}{\text{R}}$$

$$= \frac{24}{6} = 4\text{ A}$$

Combine Equation (16.29) and Equation (16.34), which gives

$$\frac{v_P}{v_S} = \frac{i_S}{i_P} \qquad (16.36)$$

Apply Equation (16.31) to find the current in the primary winding

	$\frac{120}{24} = \frac{4}{i_P}$ $i_P = 0.8\text{ A}$	

Example 16-8

A 400-turn winding on a magnetic core is excited by a 60-Hz sinusoidal primary voltage of 200 V (RMS). Find the maximum flux density in the core if the uniform cross-sectional area of the core is 10 cm × 10 cm. Ignore the resistance of the winding and any leakage flux.

Solution: Since the resistance of the winding is zero, the voltage drop in the winding is ignored. Accordingly $e_P = V_P = 400$ V. Apply Equation (16.33) to find the maximum flux

	$\phi_{\max} = \frac{200}{4.44 \times 60 \times 400} = 0.0018\text{ Wb}$	

The corresponding flux density is given by

	$B_{\max} = \frac{0.0018}{0.1 \times 0.1} = 0.18\text{ Wb/m}^2$	

16.7.2 Modeling a Real Transformer

In electrical engineering, it is generally important to use an equivalent circuit model to describe the non-ideal operation of a device such as a transformer. While an ideal model may be well suited for rough approximations, the non-ideal parameters are needed for careful transformer circuit designs. Knowing the non-ideal parameters allows the engineer to optimize a design using equations rather than inefficiently testing physical implementations in the lab.

Equations presented in the previous section represent the ideal case, which do not completely represent the physical nature of transformers. A number of loss mechanisms should be included in a practical transformer model. These losses include ohmic losses at the primary and secondary coils due to the resistance of the wires that form the coils, and the effects of the leakage flux for various magnetic core losses. The nonlinear nature of the ferromagnetic material (the dependence of permeability on magnetic flux intensity) further compounds the difficulty of an exact modeling of real transformer.

Copper (I^2R) loss. Heating losses in the resistance of primary and secondary windings are called copper losses. The resistances of the primary and secondary windings are modeled in the transformer equivalent circuit by R_P and R_S, in series with the primary and secondary of the ideal transformer. They are proportional to the square of the current in the windings. The flow of the magnetizing current through the resistance of the winding does create a real $I^2 R$ loss and voltage drop, although both are generally quite small.

Eddy Current Loss. Iron losses are due two effects. First is the *eddy current*, which is induced by the time-varying magnetic flux flows in the ferromagnetic core in accordance with Faraday's law. The flow of these currents will generate local heating due to ohmic power loss in the resistance of the core. As a matter of fact, this is the principle of induction heating. In transformers, using core materials that have high permeability but low conductivity (high μ and low σ) can reduce these eddy currents. Also, fabricating the magnetic structure out of thin insulated laminations can reduce these losses.

Hysteresis Loss. If an AC voltage is connected to a magnetizing coil, an AC magnetomotive force (MMF) causes the magnetic domains to be constantly reoriented along the magnetizing axis. The molecular motion produces heat. The power loss due to this heat is called hysteresis loss. This type of loss represents the work, done in the iron core to cyclically reorient the magnetic domains. Hysteresis losses are proportional to the area of the hysteresis loop, volume of the iron core and the frequency of the flux.

Core loss. Hysteresis and eddy current losses are collectively known as *core loss* or core loss (even if a ferrite core is used). The iron loss varies with the flux ϕ and accordingly is proportional to the input voltage. A resistance in parallel in the equivalent circuit models these iron losses.

Flux Leakage. Some flux escapes the magnetic structure and therefore fails to couple the primary and secondary. This flux produces a self inductance in the primary and secondary windings. Inductors L_P and L_S, in series with the primary and secondary resistances model the energy associated with this escaped flux. In an AC equivalent circuit, these inductances become primary and secondary leakage reactances, X_P and X_S.

Exciting current. The magnetic structure (core) needs a current in order to be magnetized. This current is called *excitation current* I_e. The excitation current remains constant from no load to full load. The in-phase component of the exciting current, I_C, supplies the core losses, and the out-of-phase component I_m, supplies the magnetic energy in the magnetic structure.

16.7.3 Transformer Equivalent Circuit

The equivalent circuit model for the non-ideal transformer is shown in Figure 16-11. An ideal transformer with resistors and inductors in parallel and series modeling the effects of winding resistances and magnetic leakage replaces the non-ideal transformer. This model is called the high side equivalent circuit model when all parameters are moved to the primary side of the ideal transformer as shown in Figure 16-12.

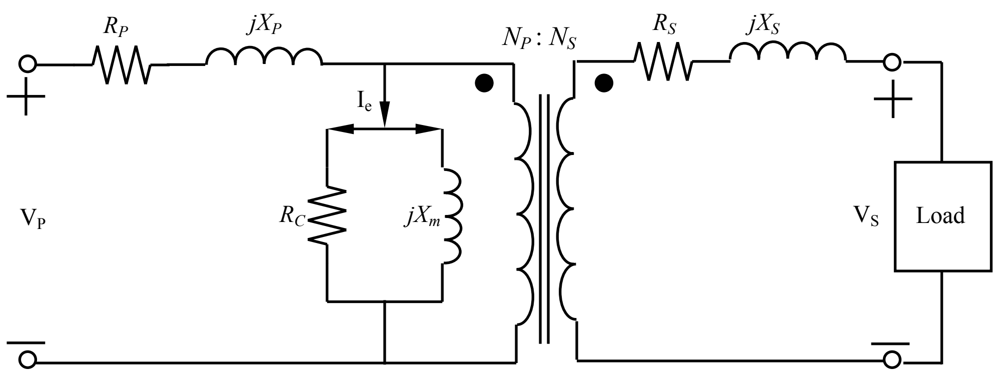
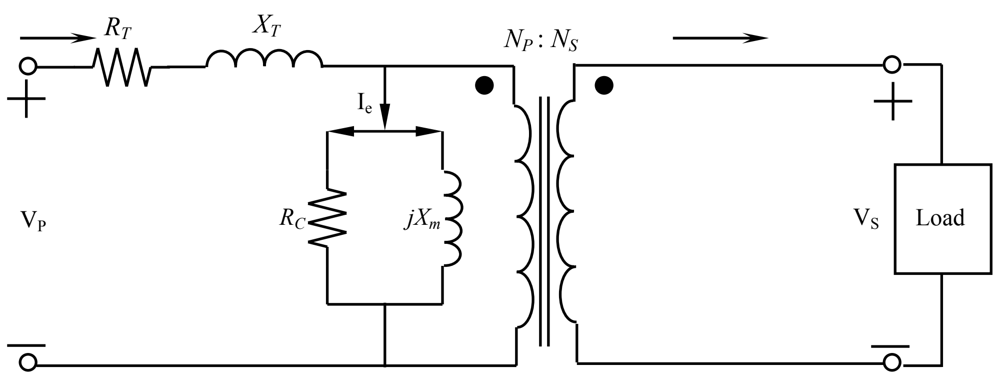


Figure 16-11 Transformer equivalent circuit.



$$R_T = R_P + \left(\frac{N_p}{N_s}\right)^2 R_S \qquad X_T = X_P + \left(\frac{N_p}{N_s}\right)^2 X_S$$

Figure 16-12 Equivalent circuit with parameters moved to the primary side.

It is easy to realize how each parameter of the equivalent circuit model could be adjusted by changing the transformer design. For example, increasing the diameter of the wire in the windings decreases the series resistance. Therefore, the equivalent circuit model parameters can be used as a way to evaluate a transformer, or compare transformers.

The parameters can be found in the same way that Thevenin equivalent circuit parameters are found: open circuit and short circuit tests. The parallel parameter values are found with no load connected to the secondary (open circuit) and the series parameter values are found with the secondary terminals shorted (short circuit). It is possible, for convenience in the lab, to make the tests on either the primary or the secondary.

16.7.4 Transformer Nameplate

Commercial transformer ratings are usually given on the nameplate, which indicates the normal operating conditions. The nameplate includes the primary-to-secondary voltage ration, frequency of operation, and apparent rated output power. Figure 16-13 shows an example of a transformer nameplate.

Turns ratio: 480:240
Frequency: 60 Hz
Apparent power: 4 kVA

Figure 16-13 Example of a transformer nameplate.

The voltage ratio can be used to determine the turns ratio, while the rated output power represents the continuous power level that can be sustained without overheating. It is important that this power be rated as the apparent power in kilovoltamperes, rather than real power in kilowatts, since a load with low power factor would still draw current and therefore operate near rated power.

An important quantity in this regard is the current. The maximum current that can be carried by a transformer winding is determined by the diameter of the wire used for the winding. If current is excessive in a winding, power in the form of heat will be dissipated. This heat may be sufficiently high to cause the insulation around the wire to break down. If this happens, the transformer may be permanently damaged.

The power-handling capacity of a transformer is dependent upon its ability to dissipate heat. If the heat can safely be removed, the power-handling capacity of the transformer may be increased. This is sometimes accomplished by immersing the transformer in oil, or by the use of cooling fins. The power-handling capacity of a transformer is measured in either the volt-ampere unit or the watt unit.

Voltage regulation. Since a real transformer has series impedance with it, the output voltage of a transformer varies with the load even if the input voltage remains constant. To compare transformers in this respect, it is necessary to define a quantity called *voltage regulation* (VR). *Full-load voltage regulation* is a quantity that compares the output voltage of the transformer at no load with the output voltage at full load. It is defined as

$$V_R = \frac{V_S^{NL} - V_S^{FL}}{V_S^{FL}} \times 100\,\% \qquad (16.37)$$

It is always good to have a small voltage regulation. For an ideal transformer, $V_R = 0$ percent.

Transformer efficiency. An important performance characteristics of a transformer, which takes losses into consideration, is its power efficiency η, defined as

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} \qquad (16.38)$$

where P_{loss} represents the losses in the transformer including copper (I^2R) loss and core loss (P_C). The hysteresis component of the core losses is generally greater than the eddy-current component. Often, the efficiency is expressed as percent efficiency by multiplying by 100.

To calculate the efficiency of a transformer at a given load, just add the losses and apply Equation (16.38). Since the output power is given by

$$P_{out} = V_S\, I_S \cos\theta$$

The efficiency of the transformer may be expressed as

$$\eta = \frac{V_S\, I_S \cos\theta}{V_S\, I_S \cos\theta + P_{Cu} + P_C} \times 100\,\% \qquad (16.39)$$

where P_{Cu} represents copper losses. Depending on its apparent power rating, the efficiency of power transformers varies from 96 to more than 99 percent.

Usually large transformers have higher efficiencies.

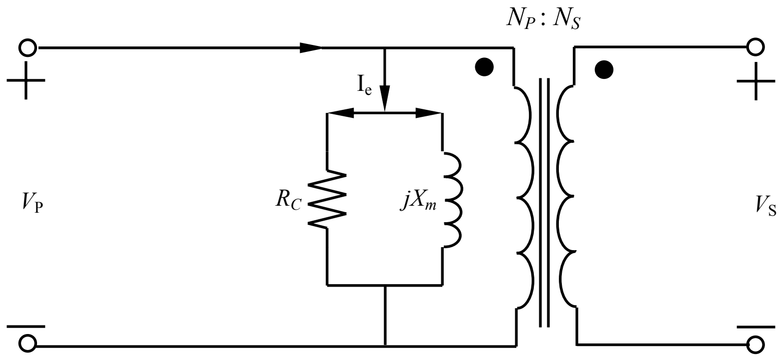


Figure 16-14 Transformer on no load.

16.7.5 Transformer on No-Load

When there is no current in the secondary winding, the transformer is said to be on no-load. Even when the transformer is on no-load, a current known as the *exciting current* (I_e) flows in the primary because of the core losses and the permeability of the core material. The exciting current has two components, the *core-loss current* (I_C) and the *magnetizing current* (I_m). The core-loss current is in phase with the induced primary voltage. It is given by

$$I_C = \frac{P_C}{V_S} \qquad (16.40)$$

Where P_C is the core loss given by the sum of the hysteresis and eddy-current losses. The case of a transformer on no-load is shown in Figure 16-14. The no-load current is given by

$$I_e = \sqrt{I_C^2 + I_m^2} \qquad (16.41)$$

And the no-load power factor is given by

$$\cos\theta = \frac{I_C}{I_e} \qquad (16.42)$$

Figure 16-15 shows the corresponding diagram.

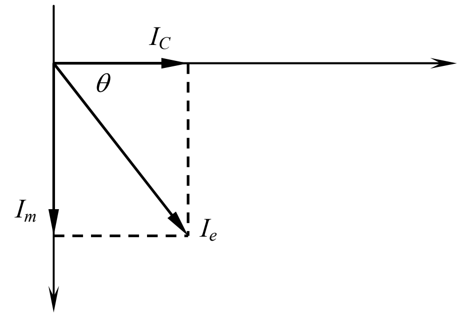


Figure 16-15 Phasor diagram for a transformer on no-load.

16.7.6 Transformer Tests

The manufacturers supply most transformers with polarity marking such as dot marking. According to the dot marking system, if the primary current is flowing into the dotted terminal of one winding, the secondary load current will be flowing out of the dotted terminal of the other winding. By applying rated voltage to one winding, the voltage across the remaining terminal is measured.

In order to find the parameters of a transformer equivalent circuit, two tests should be conducted: the open-circuit (no-load) test, and the short-circuit test.

Open-circuit test. The open-circuit test is utilized to determine the no-load current, called the *exciting current* I_e and its associated parameters, R_i and X_i. Usually the excited current varies between 1 and 6 percent of rated current in power transformers. The transformer is excited at rated voltage on the secondary side, V_S for safety in testing and instrumentation. The primary side should be covered with insulating material to prevent accidental contact. Since no load is connected to the secondary, the copper losses in the secondary are zero, and the copper losses in the primary are negligible.

Figure 16-16 (a) shows the equivalents circuits for the open-circuit test. The corresponding phasor diagram for the exciting current is shown in Figure 16-16 (b). The exciting current can be divided into two right-angle components: a core-loss component that supplies the hysteresis and eddy-current losses in the iron, and a magnetizing component that establishes the mutual flux that links both primary and secondary windings.

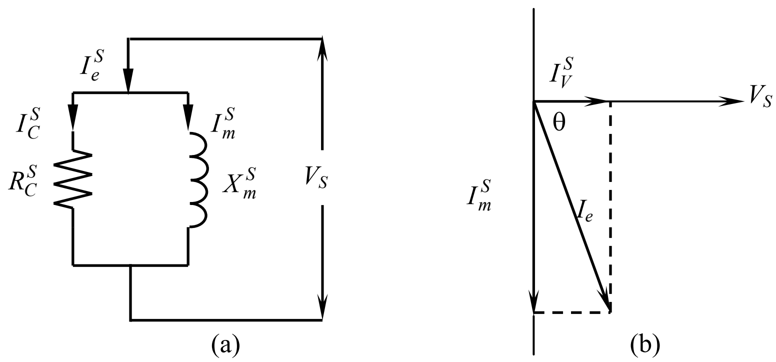


Figure 16-16 (a) Equivalent circuit for open-circuit test. (b) Phasor diagram showing no-load conditions.

The voltage V_S, exciting current in the secondary circuit I_e^S, and power in the secondary circuit P_C^S allows determination of R_C^S and X_C^S (both are referred to the secondary circuit of the transformer). The power indicated in this test is the magnetic material (iron) loss only because copper losses are very small.

In order to find P_C^S, R_C^S, and X_C^S, we should find I_C^S, and I_m^S as

$$I_C^S = \frac{P_C^S}{V_S}, \text{ and} \qquad I_C^S = \sqrt{\left(I_e^S\right)^2 - \left(I_C^S\right)^2} \tag{16.43}$$

The circuit parameters are calculated according to

$$R_C^S = \frac{V_S}{I_C^S}, \text{ and} \qquad X_m^S = \frac{V_S}{I_m^S} \tag{16.44}$$

Example 16-9

An 8-kVA, 1600/240-V, 60-Hz, single-phase transformer has an open-circuit test excited on the secondary side of the circuit (240 V). The measured exciting current is 1.0 A and the measured power is 150 W. Find R_C^S and X_m^S.

Solution: Use Equation (16.43)

$$I_C^S = \frac{150}{240} = 0.625\text{ A}$$

$$I_m^S = \sqrt{(1.0)^2 - (0.625)^2} = 0.78\text{ A}$$

Then use Equation (16.44)

$$R_C^S = \frac{240\text{ V}}{0.625\text{ A}} = 384\,\Omega\text{, and}$$

$$X_m^S = \frac{240\text{ V}}{0.78\text{ A}} = 307.7\,\Omega$$

Short-circuit test. The purpose of short-circuit test is to determine the equivalent resistance, equivalent leakage reactance, and equivalent impedance of the transformer windings. The short-circuit equivalent circuit is shown in Figure 16-17.

The total winding resistance, R_T and the total leakage reactance, X_T in Figure 16-12 are computed by shorting the low voltage winding and exciting the primary side at reduced voltage to produce the rated primary current, I_P. The primary voltage, V_{SC}, current, I_{SC}, and power P_{SC} are measured. Then R_T and X_T are calculated.

$$R_T = \frac{P_{SC}}{I_{SC}^2} \qquad (16.45)$$

and

$$X_T = \sqrt{\left(\frac{V_{SC}}{I_{SC}}\right)^2 - R_T^2} \qquad (16.46)$$

It is important to know that short-circuit test is related to the copper loss of the transformer under rated conditions.

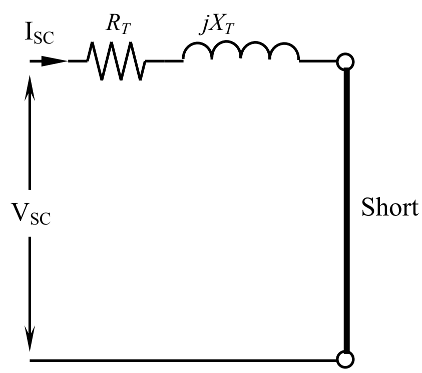


Figure 16-17 Equivalent circuit for short-circuit test.

Example 16-10

An 8-kVA, 1600/240-V, 60-Hz, single-phase transformer has the secondary side shorted. It has been found that 60.0 V on the 1600-V winding produces the rated current of 4.0 A with an input power of 150 W. Find R_T and X_T.

Solution: Use Equation (16.45)

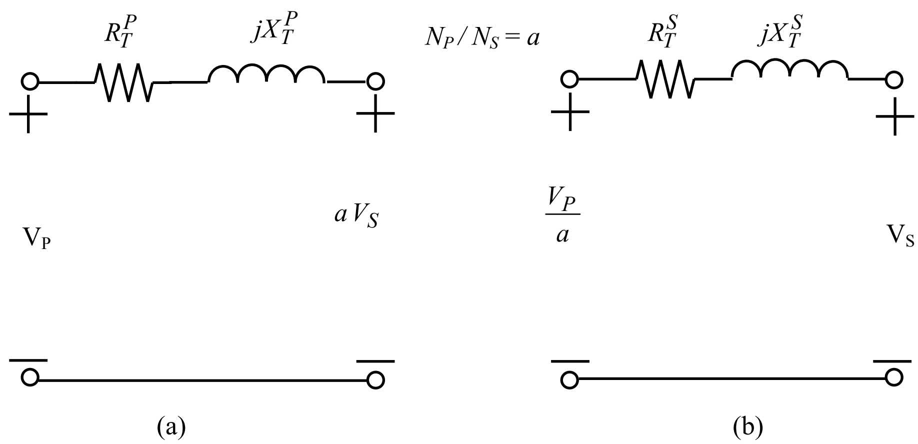
$$R_T = \frac{150\text{ W}}{(4.0)^2} = 9.375\,\Omega$$

Also, use Equation (16.46)

$$X_T = \sqrt{\left(\frac{60.0}{4.0}\right)^2 - (9.375)^2} = 11.7\,\Omega$$

16.7.7 Transformer Phasor Diagram

It is necessary to understand the voltage drops within a transformer in order to determine its voltage regulation. Consider the simplified transformer equivalent circuit shown in Figure 16-18. The effects of the excitation branch on transformer voltage regulation may be ignored, so only the series impedances need be considered. The voltage regulation of a transformer depends both on the magnitude of the series impedance and on the phase angle of the current flowing through the transformer. The easiest way to realize that is through a *phasor diagram*.



$$R_T^P = R_P + a^2 R_S \qquad R_T^S = \frac{R_P}{a^2} + R_S$$

$$X_T^P = X_P + a^2 X_S \qquad X_T^S = \frac{X_P}{a^2} + X_S$$

Figure 16-18 (a) Transformer equivalent circuit referred to the primary side. (b) Transformer equivalent circuit referred to the secondary side.

Consider the phasor diagram shown in Figure 16-19. The phasor voltage V_S is assumed to be at an angle of 0^o, and all other voltages and currents are compared to that reference. By applying Kirchhoff's voltage low (KVL) to the equivalent circuit in Figure 16-18 (b), we get

$$\frac{V_P}{a} = V_S + R_T \, I_S + jX_T \, I_S \qquad (16.47)$$

Figure 16.19 shows a phasor diagram of a transformer operating at a lagging power factor (inductive load). A phasor diagram at unity power factor (resistive load) is shown in Figure 16.20. In this case the voltage regulation is less than that of a transformer operating at lagging power factor. If the secondary current is leading (capacitive load), the secondary voltage can actually be higher than the referred primary voltage as shown in Figure 16-21. In this case, the voltage regulation will be negative.

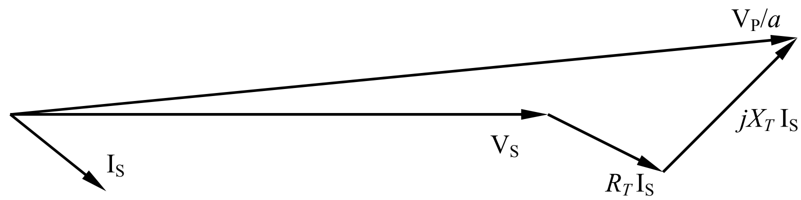


Figure 16.19 Phasor diagram of a transformer operating at a lagging power factor.

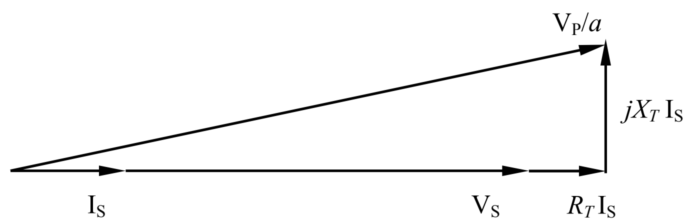


Figure 16.20 Phasor diagram of a transformer operating at unity power factor.

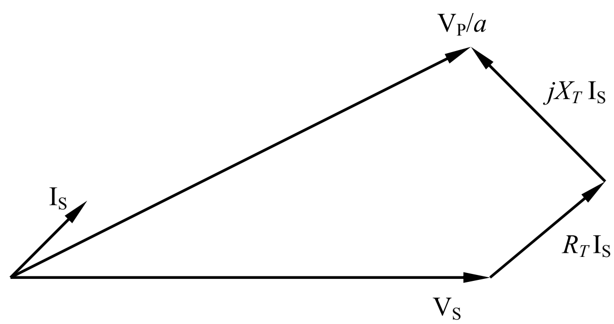


Figure 16-21 Phasor diagram of a transformer operating for a leading power factor.

Example 16-11

An open-circuit (OC)/short circuit (SC) test is performed on a single-phase transformer. The results are shown in the following table:

Quantity	OC Primary	OC Secondary	SC Primary	SC Secondary
Voltage (V)	6000	240	350	0, NM*
Current (A)	0, NM	3.00	2.50	NM
Power (W)	NM	220	200	NM

- NM: not mentioned

a) What is the transformer apparent power?
b) Draw the equivalent circuit for the transformer.
c) Find the turns ratio.
d) Find the 8-circuit parameters referred to the primary and secondary side.
e) Find the magnetizing current (I_m^S) if excited on the secondary side.
f) Estimate the transformer efficiency if supplying full power at power factor 0.9.

Solution:

a) Transformers are rated for a specific apparent power. The rated apparent power is the operating level that may be sustained under a worst-case thermal environment. It is computed as

$$\text{Apparent power} = V_{oc} \times I_{sc} = 6000\,\text{V} \times 2.5\,\text{A} = 15\,\text{kVA}$$

b) Figure 16-22 shows the equivalent circuit of the transformer

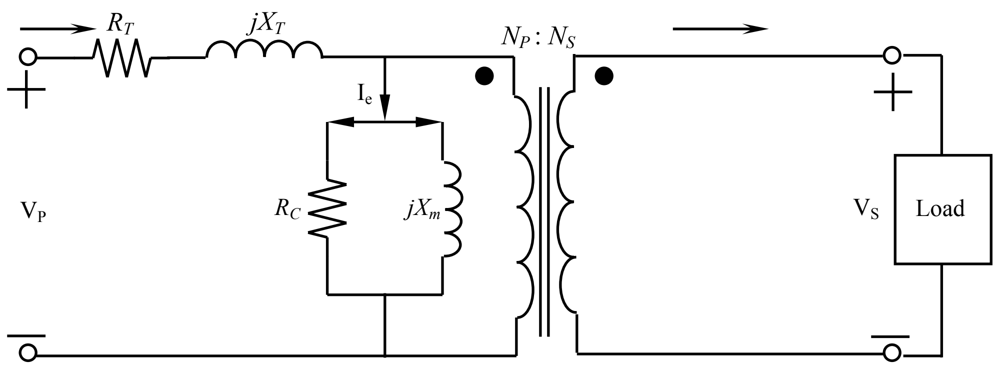


Figure 16-22 Equivalent circuit for Example 16-11.

c) The turns ratio is derived from the primary and secondary voltages. Apply Equation (16.29)

$$\frac{6000}{N_P} = \frac{240}{N_S}$$
$$a = \frac{N_P}{N_S} = \frac{6000}{240} = \frac{25}{1}$$

d) From the open-circuit test (secondary side). Apply Equation (16.43) to find the core-loss current (I_C^S)

$$I_C^S = \frac{220 \text{ W}}{240 \text{ V}} = 0.916 \text{ A}$$

Apply Equation (16.44) to find R_V^S

$$R_C^S = \frac{240 \text{ V}}{0.916 \text{ A}} = 262\,\Omega$$

Use Equation (16.35) to compute R_C referred to the primary side

$$R_C^P = 262 \times 25^2 = 163.75\text{ k}\Omega$$

Apply Equation (16.41) to find the magnetizing current (I_m^S)

$$I_m^S = \sqrt{(3.0)^2 - (0.916)^2} = 2.85 \text{ A}$$

Use Equation (16.44) to find X_m^S

$$X_m^S = \frac{240 \text{ V}}{2.85 \text{ A}} = 84.21\,\Omega$$

Use Equation (16.35) to compute X_m referred to the primary side

$$X_m^P = 84.21 \times 25^2 = 52.63\ \text{k}\Omega$$

From the short-circuit test, apply Equation (16.45) to find the total equivalent resistance R_T referred to the primary side

$$R_T = \frac{200\ \text{W}}{2.5^2\ \text{A}} = 32\ \Omega$$

and R_T referred to the secondary side is

$$R_T = \frac{32}{252} = 0.0512\ \Omega$$

Use Equation (16.46) to find X_T

$$Z_T = \frac{350\ \text{V}}{2.5\ \text{A}} = 140\ \Omega$$

$$X_T = \sqrt{(140)^2 - (32)^2} = 136.3\ \Omega$$

X_T referred to the secondary side is

$$X_T = \frac{136.3}{25^2} = 0.218\ \Omega$$

e) The magnetizing current I_m, if excited to the primary side is

$$I_m^P = \frac{6000\ \text{V}}{52.63\ \text{k}\Omega} = 0.114\ \text{mA}$$

f) Apply Equation (16.39) to compute the efficiency

$$\eta = \frac{15000 \times 0.9}{15000 \times 0.9 + 220 + 200} = 0.97 = 97\%$$

16.8 THREE-PHSAE TRANSFORMERS

Up to this point, we have focused essentially upon single-phase transformers. Single-phase means two power lines as an input source; therefore, only one primary and one secondary winding is required to accomplish the voltage transformation. However, most power is distributed in the form of three-phase AC.

Before proceeding, we should understand what is meant by three-phase power. Basically, most power generating plants produce electricity by rotating three coils or windings through a magnetic field within the generator. These coils or windings are spaced 120^o apart. As they rotate through the magnetic field they generate power, which is then sent out on three power lines. A bank of three identical single-phase transformers suitably connected or three-phase transformers with three coils or windings connected in the proper sequence must be employed.

Three-phase electricity powers large industrial loads more efficiently than single-phase electricity. When single-phase electricity is need, It is available between any two phases of a three-phase system, or in some systems, between one of the phases and ground. By the use of three conductors a three-phase system can provide 173% more power than the two conductors of a single-phase system. Three-phase power allows heavy duty industrial equipment to operate more smoothly and efficiently. Three-phase power can be transmitted over long distances with smaller conductor size.

In a three-phase transformer, there is a three-legged iron core as shown in Figure 16-23. Each leg has a respective primary and secondary winding. The three primary windings are connected to provide the proper sequence (or correct polarity) required and will be in a configuration known as "delta (Δ) or wye (Y)" The three secondary windings are connected to provide the proper sequence (or correct polarity) required. However, the secondary windings, depending on voltage requirements, will be in either "delta" or a "wye" configuration. In general, four possible combinations of connection for the three-phase, two-winding transformers are:

Y-Δ	Δ-Y	Δ-Δ	Y-Y

The current and voltage relationships between phase and line values for the Y connection are

$$V_{\text{line}} = \sqrt{3}\,.V_{\text{phase}}$$
$$I_{\text{line}} = I_{\text{phase}} \qquad (16.48)$$

The current and voltage relationships between phase and line values for a Δ connection are

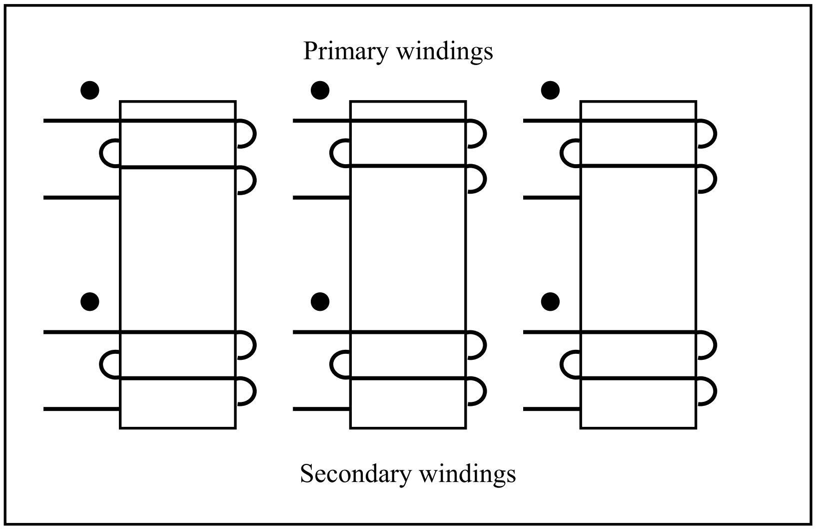


Figure 16-23 A three-phase transformer.

	$I_{\text{line}} = \sqrt{3}\,.\,\text{I}_{\text{phase}}$ $V_{\text{line}} = \text{V}_{\text{phase}}$	(16.49)

A three-phase transformer for a given rating, compared to a bank of three single-phase transformers, costs less, weights less, and requires less space. In addition, the number of external connections is reduced from twelve to six, and so a other related parts.

The key to analyzing any three-phase transformer bank is to look at a single transformer in the bank. Any single transformer in the bank behaves exactly like the single-phase transformer already studied in Section 16.7. All the quantities calculations for a three-phase transformer is done on a per-phase basis using the same techniques already discussed for single-phase transformer.

Example 16-12

A 100 kVA bank of Y-Δ-connected step-down transformers has an input line voltage of 4160-V and an output line voltage of 240-V. Determine

a) Bank ratio.
b) Transformer ratio.

c) Rated line and phase currents for the primary side.
d) Rated line and phase currents for the low side.

Solution:

a) The bank ratio is the ratio of high-side voltage to low-side voltage

$$\frac{V_{line}(HS)}{V_{line}(LS)} = \frac{4160}{240} = 17.3$$

b) The transformer ratio is the ratio of phase voltages. For the Y primary

$$V_{line} = \sqrt{3}\, V_{phase}$$

$$V_{phase} = \frac{4160}{\sqrt{3}} = 2402 \text{ V}$$

For the secondary

$$V_{phase} = V_{line} = 240 \text{ V}$$

$$\frac{V_{phase}(HS)}{V_{phase}(LS)} = \frac{2402}{240} = 10.0$$

c) The power is calculated as

$$S = \sqrt{3} \times V_{line} \times I_{line}$$

$$I_{line} = \frac{100{,}000}{\sqrt{3} \times 4160} = 13.87$$

Since the high side is Y connected,

$$I_{phase} = I_{line} = 13.87$$

d) The power is calculated as

$$S = \sqrt{3} \times V_{\text{line}} \times I_{\text{line}}$$

$$I_{\text{line}} = \frac{100{,}000}{\sqrt{3} \times 240} = 240.66 \text{ A}$$

Since the low side is Δ connected,

$$I_{\text{phase}} = \frac{240.66}{\sqrt{3}} = 138.94 \text{ A}$$

16.9 THE PER UNIT SYSTEM FOR TRANSFORMERS

A per unit system, a system of dimensionless parameters, is used for computational convenience and for comparing the performance of a set of transformers or other types of electric machines. Per unit quantities simplify analysis of complex power systems involving transformers. When expressed in a per unit system, the parameters of transformers lie in a reasonably narrow numerical range.

In the per unit system, the voltages, currents, powers, impedances, and other related quantities are not measured in their usual SI units (volts, amperes, watts, ohms, etc.). Instead, each electrical quantity is measured as a decimal fraction of some base level. Dividing one physical parameter carries out this process of normalization by another of the same parameter. The denominator is referred to as the *base-value quantity*.

Any quantity can be expressed on a per-unit (pu) basis by the equation

$$\text{Quantity per unit (pu)} = \frac{\text{Actual value}}{\text{Base value of quantity}} \tag{16.50}$$

Actual value means a value in volts, amperes, ohms, etc. Usually, two base quantities VA_{base} and voltage V_{base} are chosen to define a given per unit system. Rest of the per-unit quantities can be calculated

	$I_{base} = \frac{VA_{\text{base}}}{V_{\text{base}}}$ $R_{base} = \frac{V_{\text{base}}}{I_{base}}$	(16.51)

Example 16-13

A single-phase, 75-kVA, 2,400:240-volt, 60 Hz distribution transformer has the following parameters: $R_P = 1.0\ \Omega$, $R_S = 0.005\ \Omega$, $X_P = 1\ \Omega$, and $X_S = 0.01\ \Omega$.

Solution: The base quantities for primary and secondary sides are given as

	Primary Side	Secondary Side
VA_{base}	75,000 VA	75,000 VA
V_{base}	2,400 V	240 V
I_{base}	$\frac{75000\text{ VA}}{2400\text{ V}} = 31.25\text{ A}$	$\frac{75000\text{ VA}}{240\text{ V}} = 312.5\text{ A}$
R_{base}	$\frac{2400\text{ V}}{31.25} = 76.8\,\Omega$	$\frac{240\text{ V}}{312.5\text{ A}} = 0.768\,\Omega$

16.10 ELECTROMECHANICAL ENERGY CONVERSION

The transformer, as explained in Section 16.7, is an electromagnetic device that transmits electrical energy with a change in voltage and current levels from one side to the other. This section deals with the principles of electromechanical energy conversion and their applications.

Electromehanical systems are devices that can convert either mechanical energy to electric energy or electric energy to mechanical energy. When a device is capable of converting electric energy to mechanical energy, it is called a motor. When it converts electric energy to mechanical energy, it is called a motor.

16.10.1 Essential Features

Mechanical forces may be converted to electrical energy, and vise versa, by means of the coupling field provided by energy stored in the magnetic field. While understanding energy conversion, it is important to understand the various energy storage and loss mechanisms in the electromagnetic field. Figure 16-24 illustrates the coupling between the electrical and mechanical systems.

In the electrical system, energy loss may occur because of resistance, while in the mechanical system, energy loss can occur because of the heat developed as a consequence of friction. Losses occur also in the magnetic coupling medium due to the presence of eddy currents and hysteresis losses.

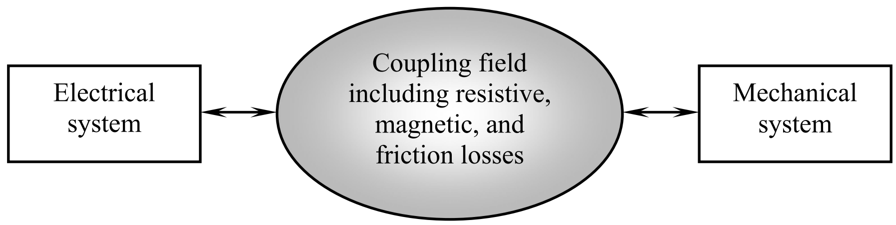


Figure 16-24 Coupling between electrical and mechanical systems.

16.10.2 Generation of Force (Motor Action)

A significant effect of a magnetic field on its surroundings is that it induces a force on a current-carrying conductor within the field as illustrated in Figure 16-25. You see a conductor of length l and current *i* emerged in a uniform magnetic field of flux density **B**, pointing into the page. The force induced on the conductor is given by

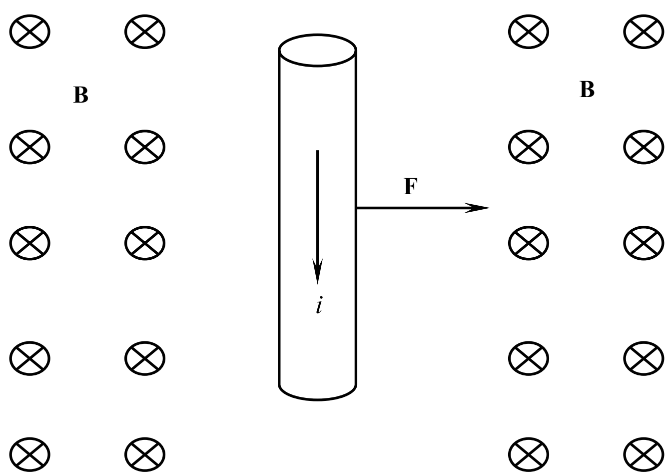


Figure 16-25 A current-carrying conductor emerged in a magnetic field.

	$\mathrm{F} = i\left(\mathrm{l} \times \mathrm{B}\right)$	(16.52)

where, i is the magnitude of current in conductor in amperes (A), l is the length of conductor in meters (m) with a direction to be in the direction of current flow, and **B** is the magnetic flux density vector in teslas (T). The force is measured in newtons (N).

The direction of the force is given by the right-hand rule: The force F generated by the current I is in the direction that would push the conducting bar to the right. The magnitude of this force is $F = Bli$ if the magnetic field and the direction of current are perpendicular. If they are not, then we must consider the angle θ formed by B and l; in the more general case

	$F = Bli\sin\theta$	(16.53)

The induction of force in a current-carrying conductor in the presence of a magnetic field is the basis of motor action.

Example 16-14

Consider the current-carrying conductor shown in Figure 16-25. The magnetic flux density is 0.5 T, directed into the page. If the wire is 2 m long and carries 1 A of current in the direction shown. What are the magnitude and direction of the force induced on the wire?

Solution: The direction of the force is given by the right-hand rule as being to the right. The magnitude is given by Equation (16.53)

	$F = ilB\sin\theta$ $= \left(1\ \mathrm{A}\right)\left(2\ \mathrm{m}\right)\left(0.5\ \mathrm{T}\right)\sin 90^{\circ}$ $= 1\ \mathrm{N}$, directed to the right	

16.10.3 Generation of Voltage (Generator Action)

The other mode of operation occurs when an external force causes a conducting bar to be displaced. This means, if a conductor with a proper orientation moves through a magnetic field, a voltage is induced in it as shown in Figure 16-26.

Positive and negative charges are forced in opposite directions in the circuit of Figure 16-26. Accordingly, a potential difference will appear across the conducting bar. This potential difference is the electromotive force, or EMF. The

EMF must be equal to the force exerted by the magnetic field. The voltage induced in the conductor is given by

	$V_{ind} = l.(v \times B)$	(16.54)

Where, v is the velocity of the conductor in meters per second (m/s). l points along the direction of the conductor toward the end making the smallest angle with respect to the vector v × **B**. The voltage in the conductor will be established so that the positive end is in the direction of the vector v × **B**. The induction of voltages in a moving conductor in a magnetic field is fundamental to the operation of generators. This is called generator action.

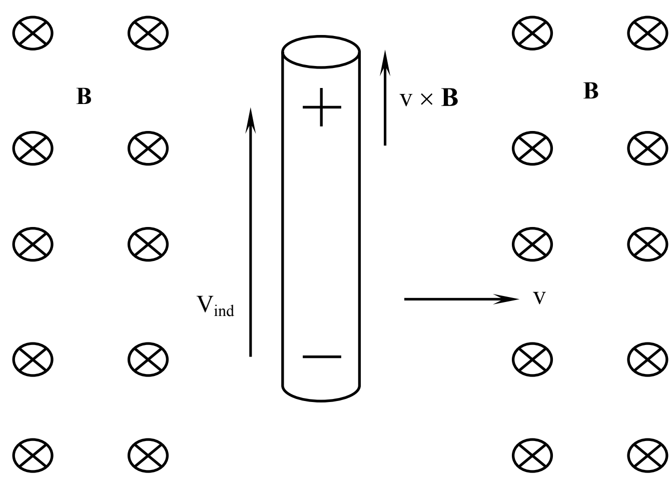


Figure 16-26 A conductor moving inside magnetic field.

Example 16-15

Consider the moving conductor with a velocity of 10 m/s to the right in the presence of a magnetic field. The flux density is 1.0 T into the page, and the conductor is 2.0 m in length. What are the magnitude and polarity of the resulting induced voltage?

Solution: The direction of the quantity v × **B** is up. Accordingly, the voltage on the conductor will be established positive with respect to the bottom of the conductor. The direction of vector l is up, so that it makes the smallest angle with respect to the vector v × **B**. Now apply Equation (16.54)

$$
\begin{aligned}
V_{ind} &= l.\left(v \times B\right) \\
&= \left(vB \sin 90^{o}\right) l \cos 0^{o} \\
&= vBl \\
&= \left(10.0\text{ m/s}\right)\left(1.0\text{ T}\right)\left(2.0\text{ m}\right) \\
&= 20\text{ V, positive at the top of the conductor}
\end{aligned}
$$

16.11 BASIC EQUATIONS FOR MACHINES

Let us consider a linear DC machine as shown in Figure 16-27. This example demonstrates how the principles of electromagnetism apply to DC machines in general. The Dc machine consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. Along the rail track is a uniform magnetic field directed into the page. A bar of conducting metal is lying across the track. The following four basic equations may be applied to the machine

1. Force on a conductor due to the presence of a magnetic field as in Equation (16.53).
2. Induced voltage on a conductor moving in a magnetic field as in Equation (16.54)
3. Kirchhoff's voltage law (KVL). We may apply KVL to the circuit in Figure 16-27.

$$V - iR - V_{ind} = 0$$

4. Newton's law for the bar across the tracks

$$F = ma \qquad (16.55)$$

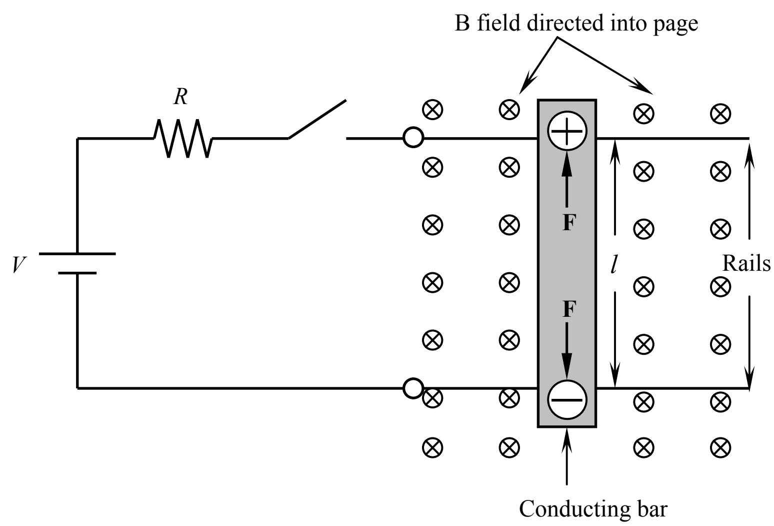


Figure 16-27 A linear DC machine.

Example 16-16

Consider the linear DC machine shown in Figure 16-27. Assume $V = 100$ V and $R = 1\ \Omega$. What is the machine's maximum starting current?

Solution: At starting condition, the velocity of the conducting bar is 0, therefore, $e = 0$. By applying KVL, the machine's starting current is

$$i = \frac{V - V_{ind}}{R} = \frac{120\text{ V} - 0\text{ V}}{1\,\Omega} = 120\text{ A}$$

SUMMARY

1. Power is the rate of doing work. Instantaneous power is the product of the voltage at the instant times the current at the instant.
2. A magnetic circuit is analogous to electric circuit and its behavior is governed by equations analogous to those for an electric circuit.
3. In an electric circuit, the applied voltage causes a current I to flow. Similarly, in a magnetic circuit, the applied electromotive (MMF) force causes flux ϕ to be produced.
4. A transformer is a device for converting electric energy at one voltage level to electric energy at another voltage level through the action of a magnetic field.
5. All values of voltage, current, and impedance in a transformer are proportional to the turns ratio.
6. The primary winding of a transformer is connected to the power supply and the secondary winding is connected to the driven load.
7. Ac step-down transformer is that has a lower secondary voltage than primary voltage.
8. A step-up transformer is that has a higher secondary voltage than primary voltage.
9. A real transformer has leakage fluxes that pass through either the primary or the secondary winding, but not both. Also, there is eddy current, hysteresis, and copper losses. These effects are modeled in the equivalent circuit of the transformer.
10. Faraday's law states that a voltage is generated in a coil of wire, which is proportional to the time rate of change in the magnetic flux passing through it. Faraday's law is the basis of transformer action.
11. A current-carrying conductor emerged in a magnetic field, if it is oriented properly, will have a force induced on it. This is the basis of motor action.
12. A conductor moving inside a magnetic field with a proper orientation will have a voltage induced on it. This is the basis of generator action.

REVIEW QUESTIONS

1. What is a ferromagnetic material? Why is the permeability of such material so high?
2. What is Faraday's law?
3. What is a transformer?
4. A transformer has a turns ratio of 1:10. The primary current is 18 A. What is the secondary current?
5. What is the leakage flux in a transformer? Why is it modeled in a transformer equivalent circuit as an inductor?
6. Describe the types of losses that occur in a transformer.
7. Why is the iron core of a transformer laminated?
8. Is it desirable to design a transformer with no leakage flux? Explain.
9. Describe the open-circuit and short-circuit tests in a transformer.
10. What are the fundamental requirements for a magnetic field to produce a force on a conductor?
11. What are the fundamental requirements for a magnetic field to produce a voltage in a conductor?

PROBLEMS

16-1 Calculate and plot the instantaneous voltage, current, and power of a 5-Ω resistor connected across a voltage source (10 V_{rms} $\angle 0^o$, 1 kHz).

16-2 Repeat Problem 16-1 for a 5-Ω reactive reactance.

16-3 Repeat Problem 16-1 for a 5-Ω capacitive reactance.

16-4 A ferromagnetic core is shown in Figure 16-28. The depth of the core is 10 cm. There is a 500-turn coil wrapped around the left side of the core. Assume a relative permeability μ_r of 3000. Draw the corresponding magnetic circuit and calculate the flux produced by a 2-A input current.

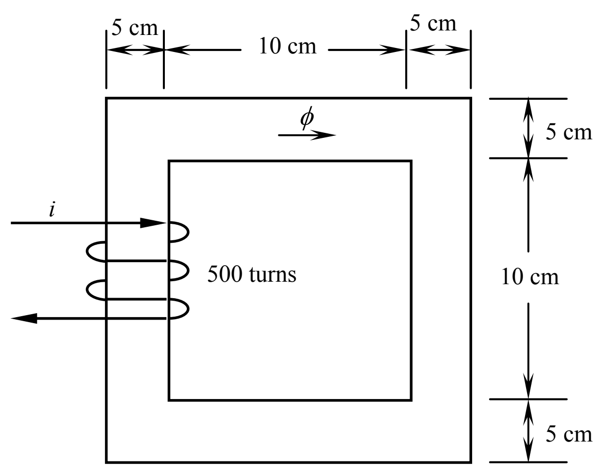


Figure 16-28 Ferromagnetic core of Problem 16-4.

16-5 A ferromagnetic core is shown in Figure 16-29. The mean path length is 50 cm. There is a small gap of 0.05 cm in the structure. The cross-sectional area of the core is 15 cm^2, the relative permeability of the core is 3000, and the coil has 500 turns. Ignore the fringing in the air gap. Find

a) The total reluctance of the flux path (core and air gap).
b) The current required to produce a flux density of 0.5 T in the air gap.

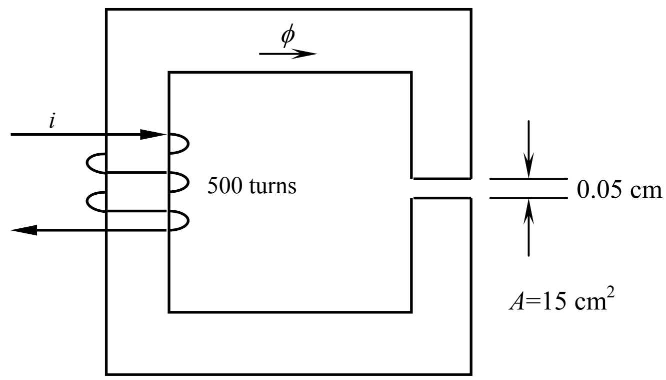


Figure 16-29 Ferromagnetic core of Problem 16-4.

16-6 An ideal transformer with a primary of 300 turns and a secondary of 30 turns has its primary connected to a 120-V, 60-Hz supply, and its secondary connected to a $200\angle 30^o\ \Omega$ load. Determine

a) The secondary voltage.
b) The load current.
c) The input current to the primary.
d) The input impedance looking into the primary terminals.

16-7 An open-circuit (OC)/short circuit (SC) test is performed on a single-phase transformer. The results are shown in the following table:

	OC	OC	SC	SC
Quantity	**Primary**	**Secondary**	**Primary**	**Secondary**
Voltage (V)	7500	240	350	0, NM*
Current (A)	0, NM	3.50	2.0	NM
Power (W)	NM	250	245	NM

NM: not mentioned

a) What is the transformer apparent power?
b) Draw the equivalent circuit for the transformer and find the 8-circuit parameters referred to the primary and secondary side.
c) Find the magnetizing current, I_ϕ, if excited on the secondary side.
d) Estimate the efficiency if supplying full VA at power factor 0.9.

16-8 A current-carrying conductor as shown in Figure 16-25 is carrying 1.0 A in the presence of a magnetic field B = 0.5 T. Calculate the magnitude of the force induced on the conductor.

16-9 A conductor moving in a magnetic field as shown in Figure 16-26. Assume its length l = 1.0 m, B = 0.3 T, into the page, v = 8 m/s, and $\theta = 45^o$. Calculate the magnitude of the induced voltage in the conductor.

16-10 Consider the DC linear machine shown in Figure 16-27. Assume B = 0.5 T directed into the page, a resistance R of 0.5 Ω, a conducting bar of length l = 1.0 m, and a battery voltage of 100 V. Calculate the initial current flow and the initial force on the bar at starting.

MULTIPLE CHOICE QUESTIONS

- The iron core of a transformer is laminated in order to:

  a) Reduce eddy currents in the core
  b) To increase the permeability of the core
  c) To reduce the electrical resistance of the core
  d) To improve the flux linkage between the two coils of the transformer.

- A transformer has a primary voltage of 240 V and turns ratio of 2.5:1. A load of 24 Ω is connected to the secondary winding. What current is flowing in the load

  a) 4 A
  b) 8 A
  c) 12 A
  d) 15 A

- A transformer is transforming 120 V in the primary circuit to 12V in the secondary circuit. If the current in the secondary circuit is 1 A, what current is flowing in the primary circuit?

  a) 10 A
  b) 100 A
  c) 12 A
  d) 0.1 A
  e) None of the above. The answer is ________________________________

- If the primary coil in question 3 has 200 windings, how many turns does the secondary coil have?

  a) 12
  b) 120
  c) 2000
  d) 20
  b. None of the above. The answer is ______________________________

- The core-loss current in a transformer is in:

  a) Phase with the induced primary voltage.
  b) Out-of-phase with the induced primary voltage.
  c) Lagging the induced primary voltage by 90^o.
  d) Leading the induced primary voltage by 90^o.

- The transformer is said to be on no-load when:

    a) There is current in the secondary.
    b) There is no current in the secondary.
    c) There are losses only in the secondary winding.
    d) There are losses only in the primary winding.

- Magnetomotive force (MMF) in a magnetic circuit is measured in

    a) ameres (A)
    b) ampere.turns (A-t)
    c) newtons (N)
    d) volts (V)